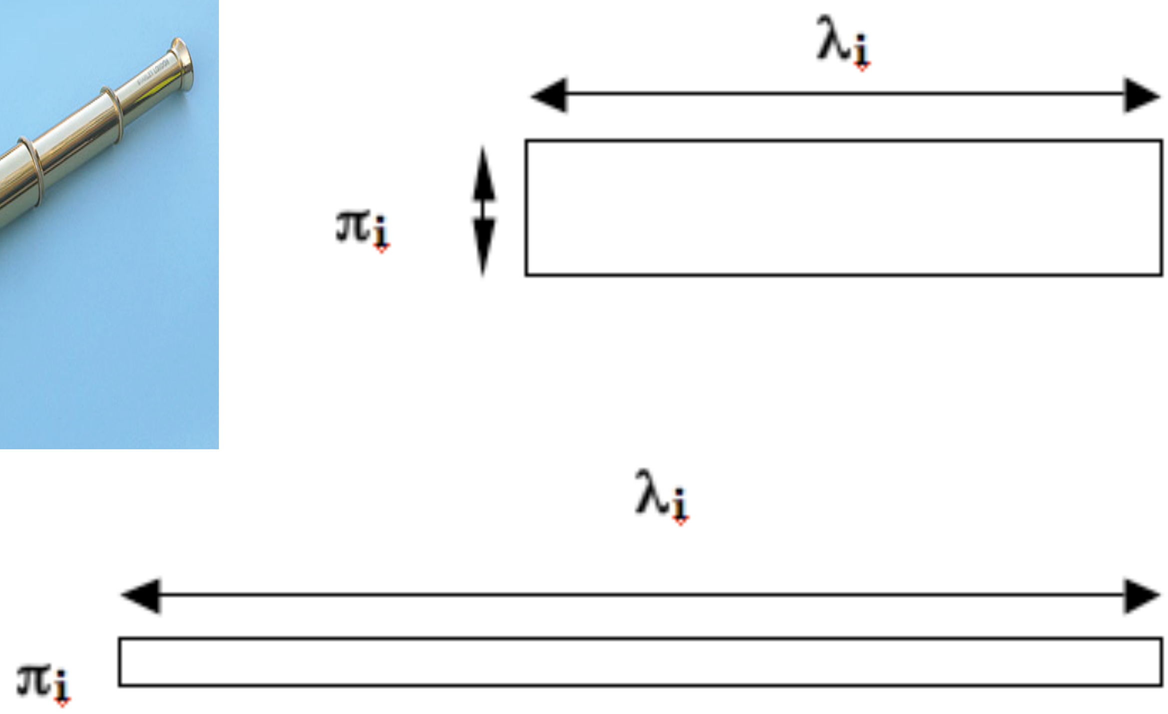
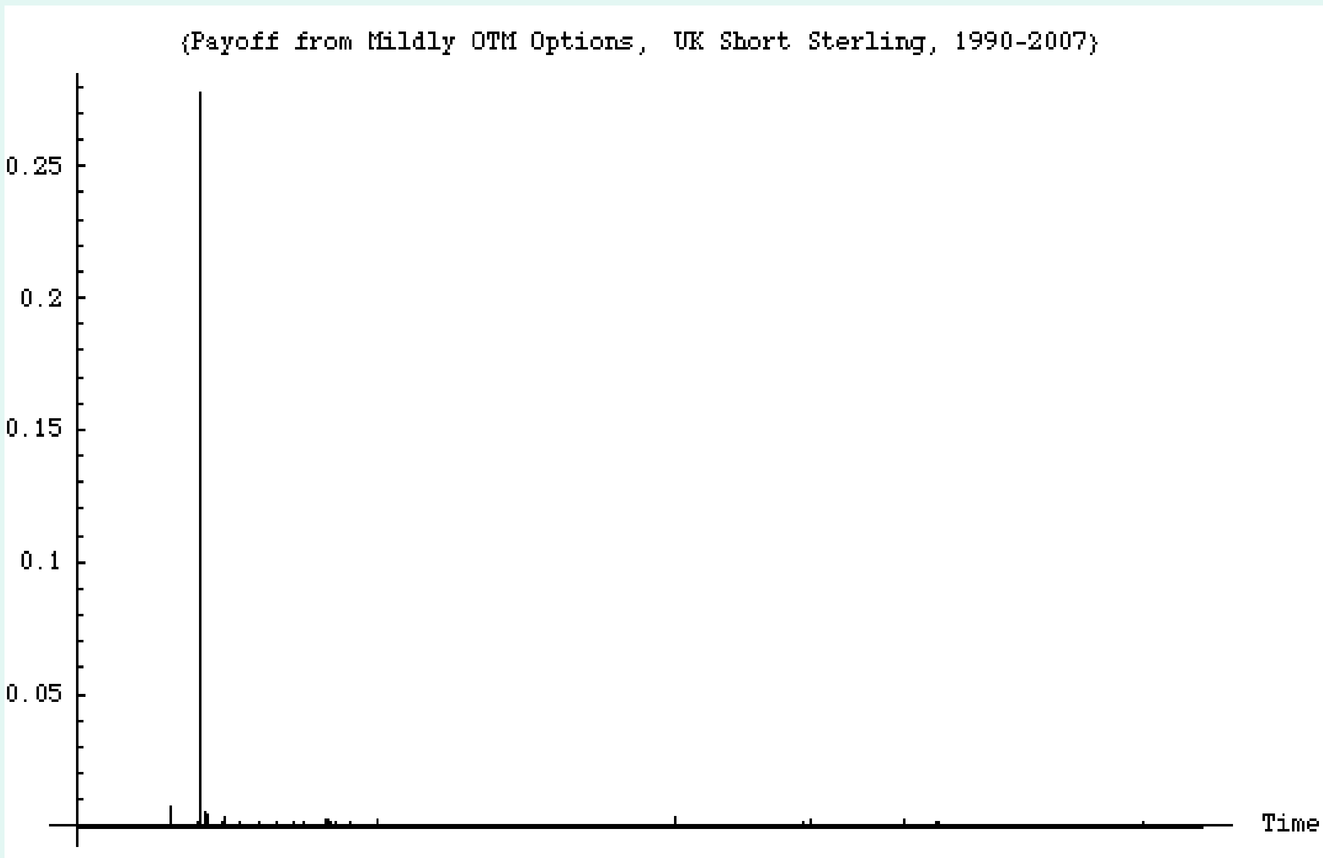
Problem of Small Probability

One 'Quant' Sees Shakeout For the Ages -- '10,000 Years'

The Telescope Problem

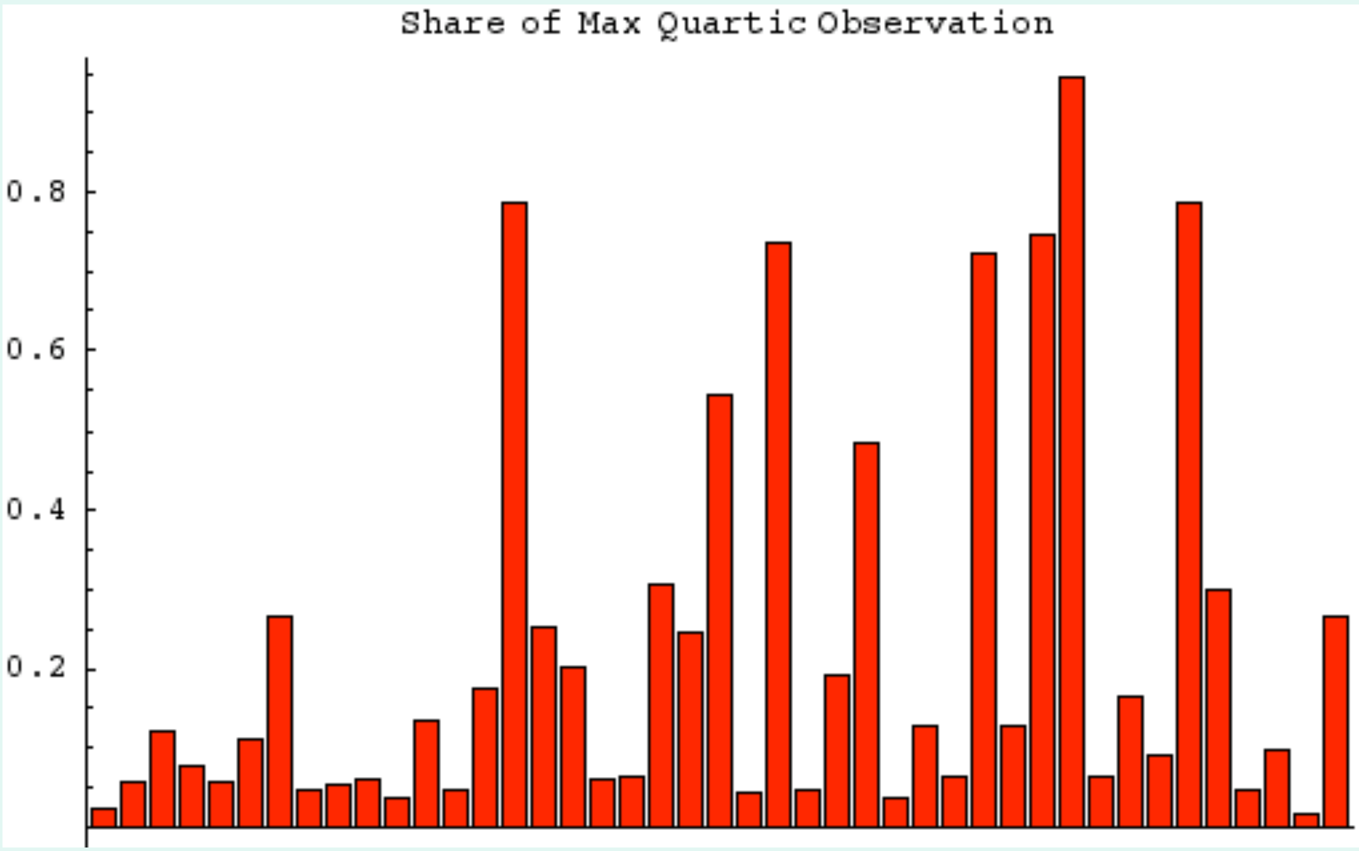


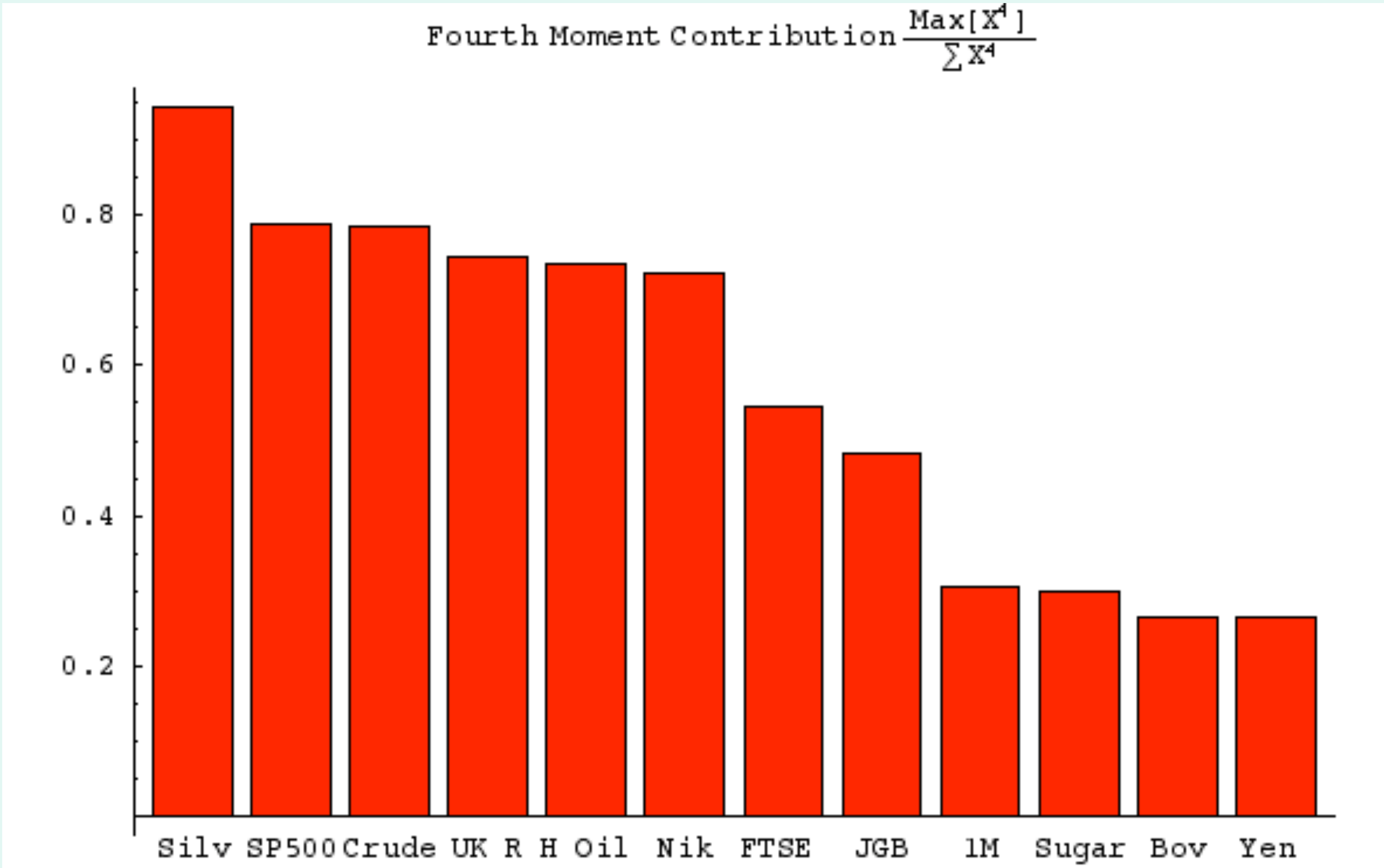

{Payoff from Mildly OTM Options, UK Short Sterling, 1990-2007}
0.25
0.2
0.15
0.1
0.05
Time

Two Domains

Type 1– CLT in real time

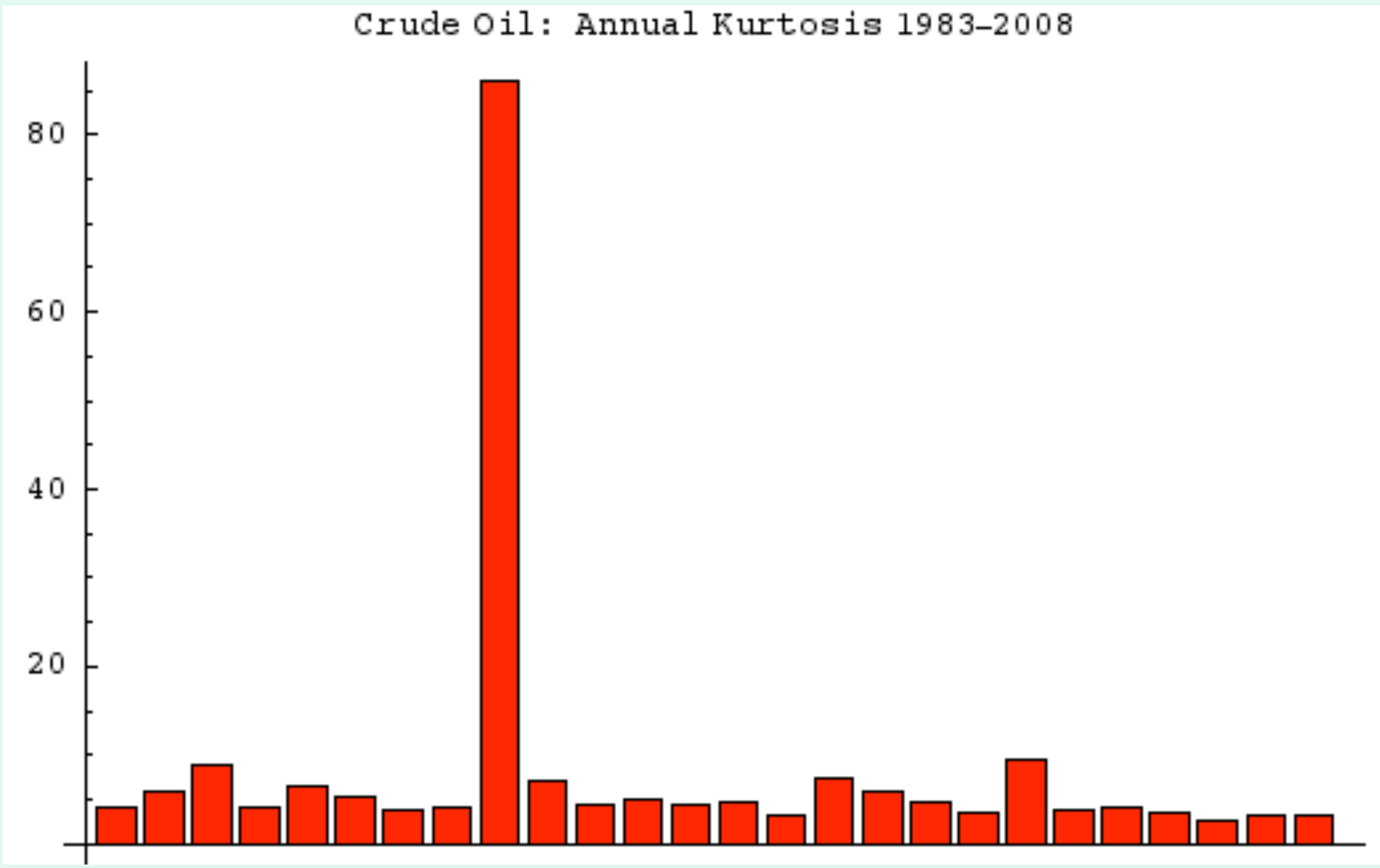
Type 2– No CLT in real time

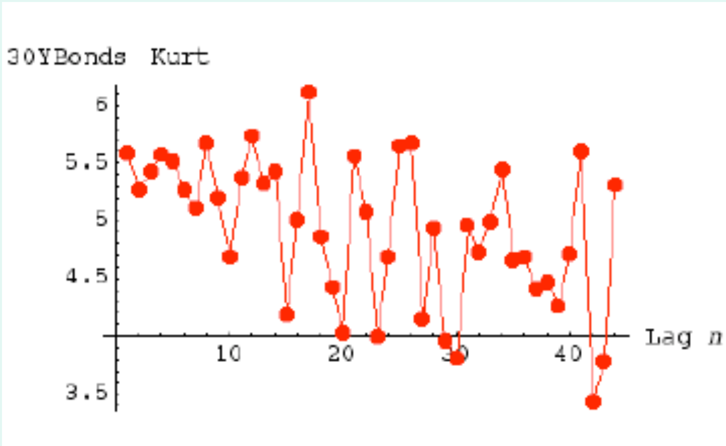

Share of Max Quartic Observation
0.8
0.6
0.4
0.2

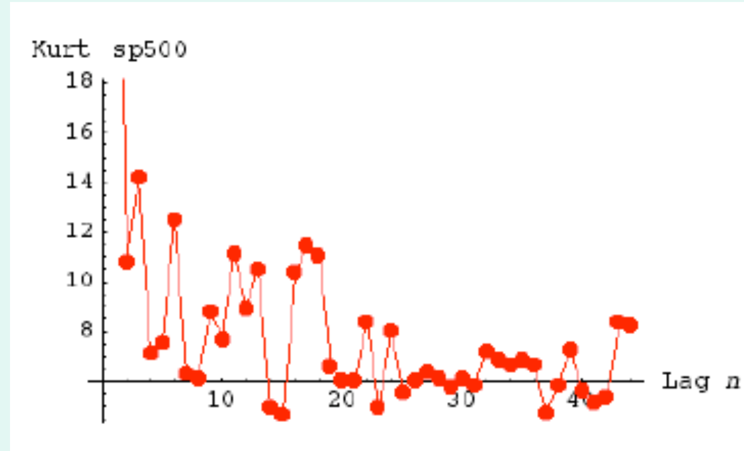


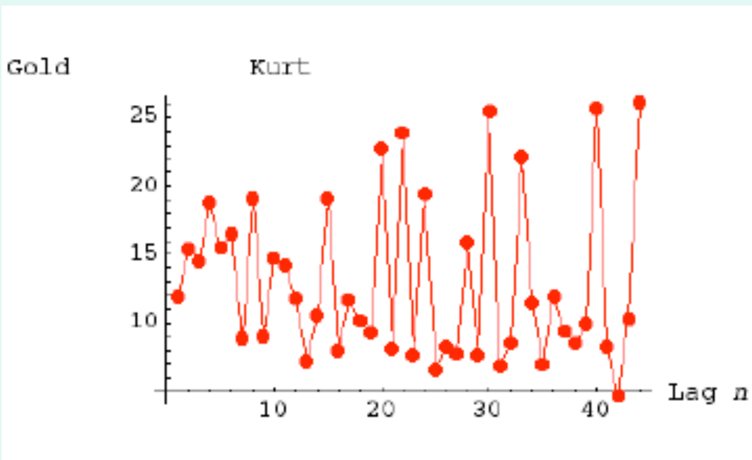
Fourth Moment Contribution $\frac{Max[X^4]}{\sum X^4}$
0.8
0.6
0.4
0.2
Silv
SP500
Crude
UK R
H Oil
Nik
FTSE
JGB
1M
Sugar
Bov
Yen

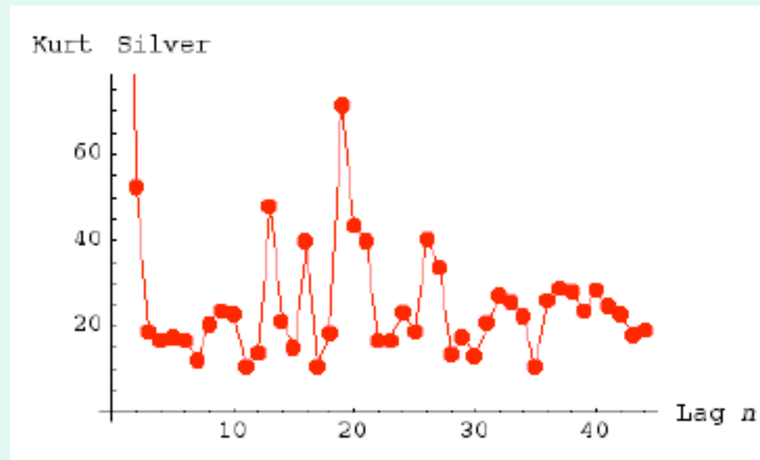
Temporal Instability




30YBonds Kurt
6
5.5
5
4.5
3.5
10
20
40
Lag n

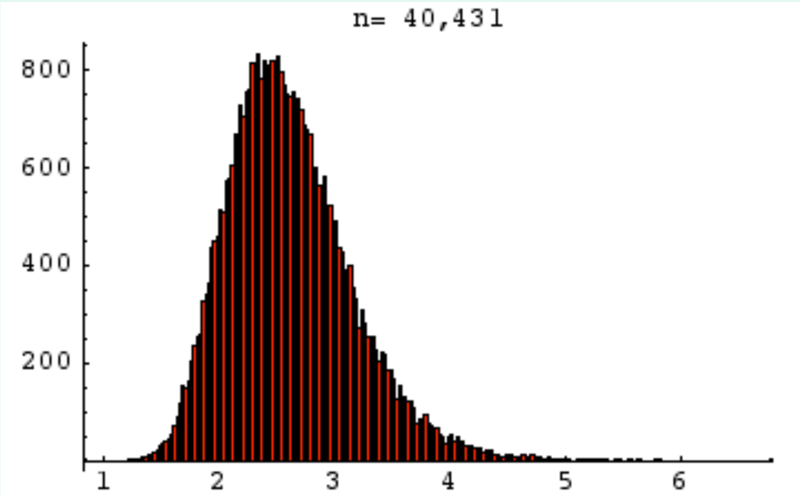

Kurt sp500
18
16
14
12
10
8
10
20
Lag n

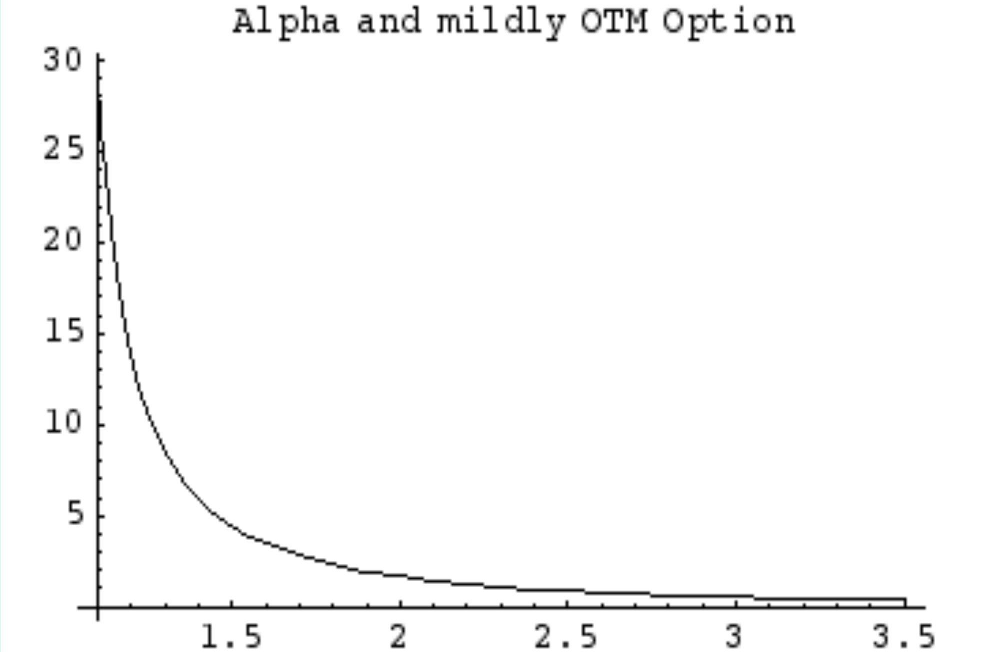

Gold
Kurt
25
20
15
10
10
20
30
40
Lag n


Kurt Silver
60
40
20
10
20
30
40
Lag n

Sampling Error & Pareto Tails

- Shortfall EXTREMELY sensitive to α





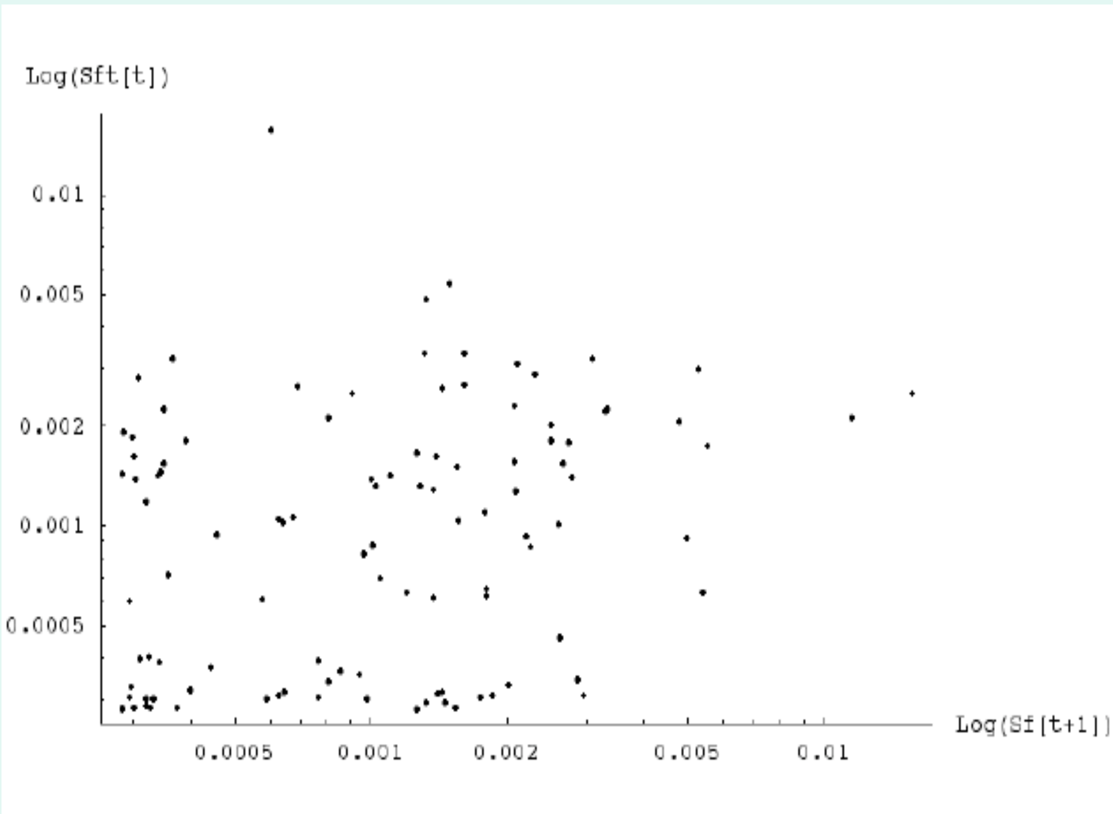
“Measurability”

- Norm L-2 : “variance” (STD), Least-Square methods, etc.
- Power Laws / scalable theoretically better, but NON-CALIBRATABLE where it matters

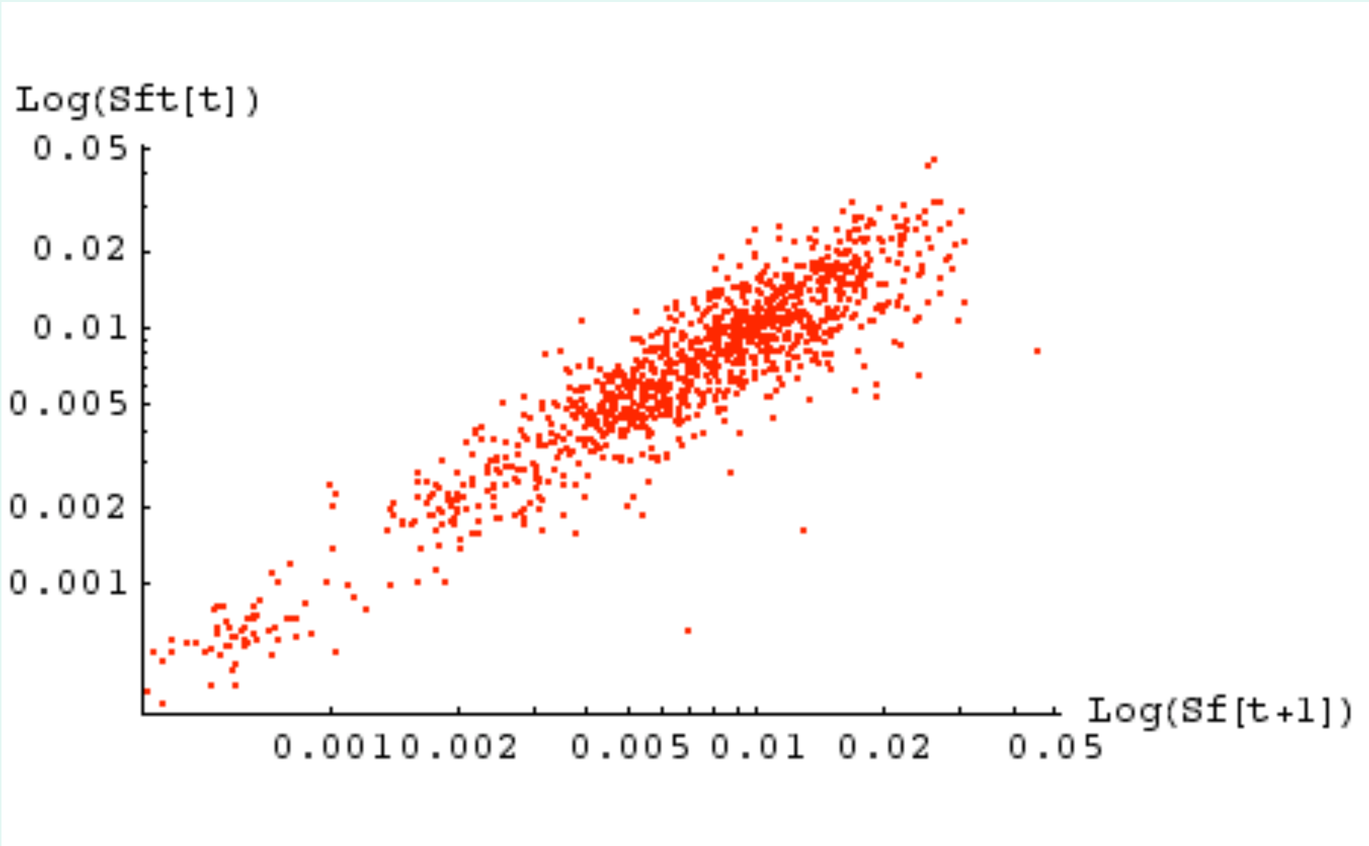
“Measurability”

- Old distinction “Knightian”, “measurable risk v/s nonmeasurable uncertainty”
- Conflation of “measure” & “estimate”
- Nonmeasurability a function of *remoteness* of the event
- Lack of rigor at the foundations
- Lack of empirical rigor

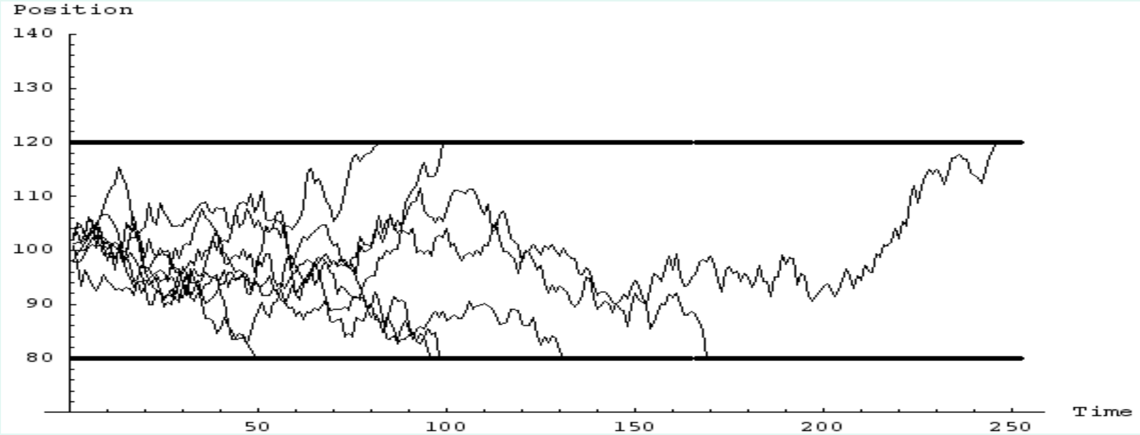
Past & Future

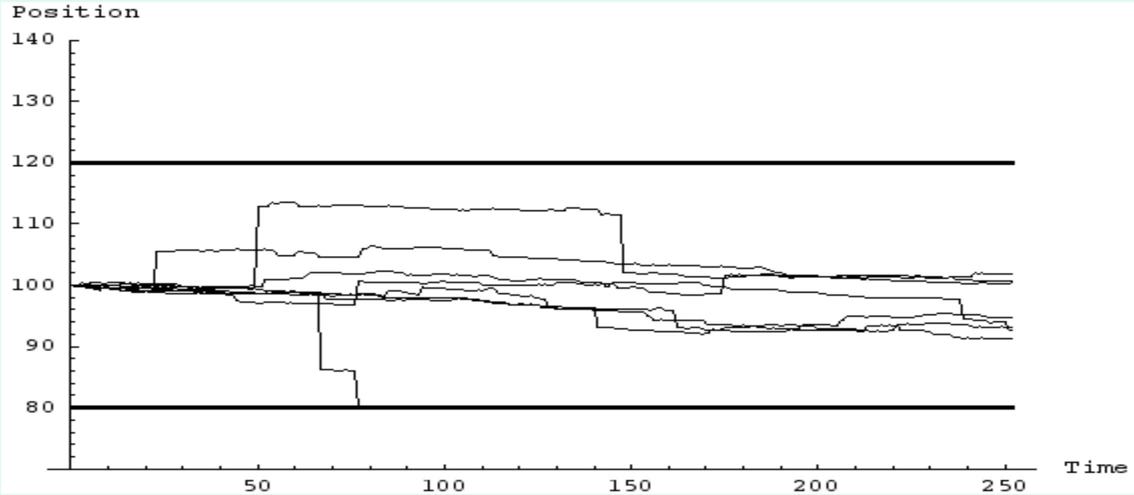

Log(Sft[t])
0.01
0.005
0.002
0.001
0.0005
Log(Sf[t+1])
0.0005
0.001
0.002
0.005
0.01

Regular is predictive of regular

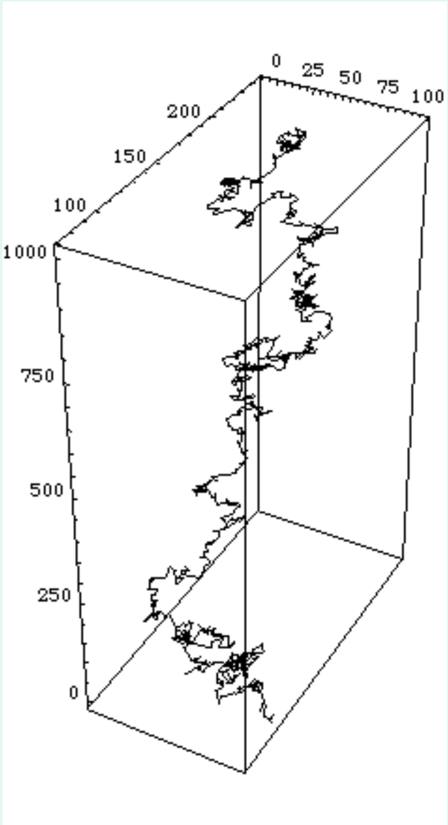


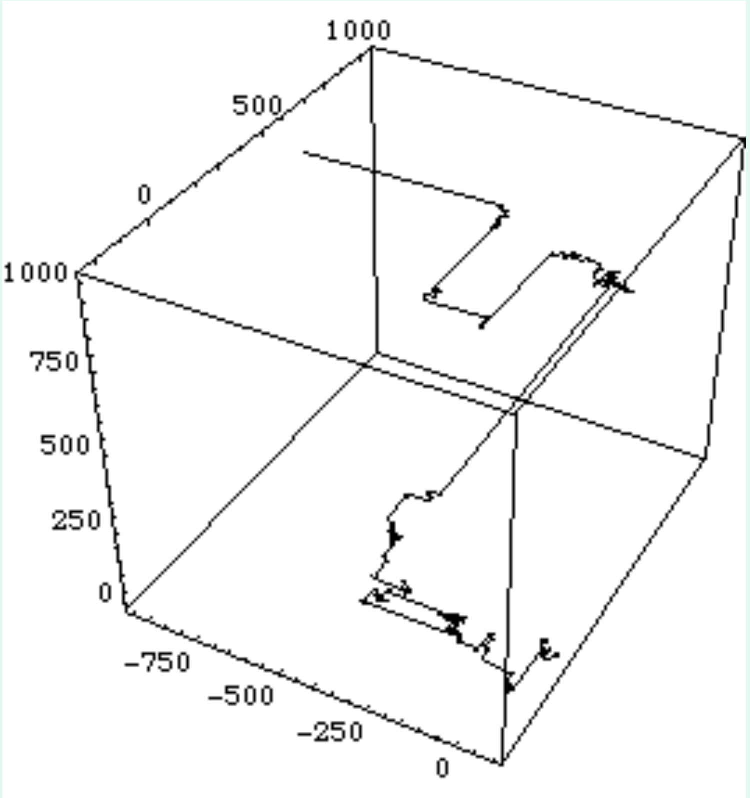
Fallacy of Volatility

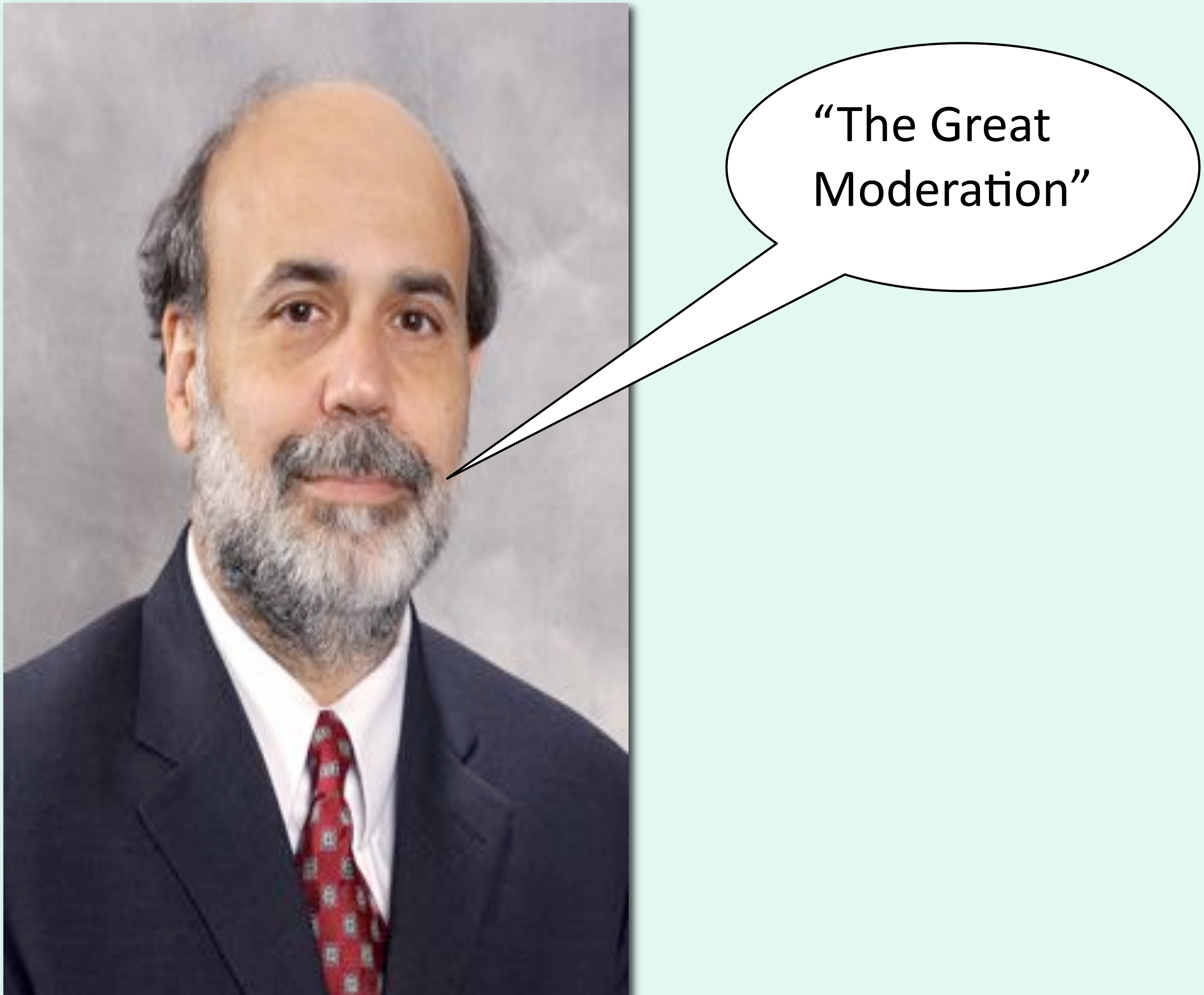


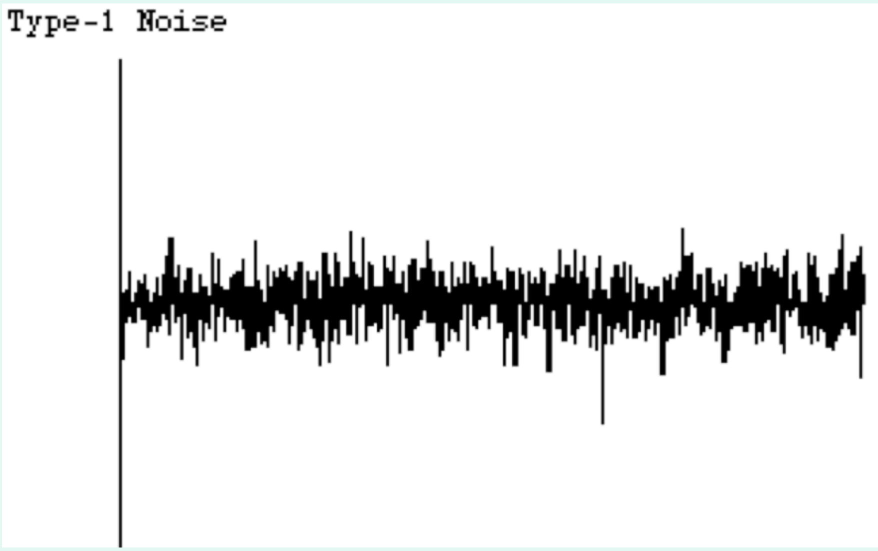


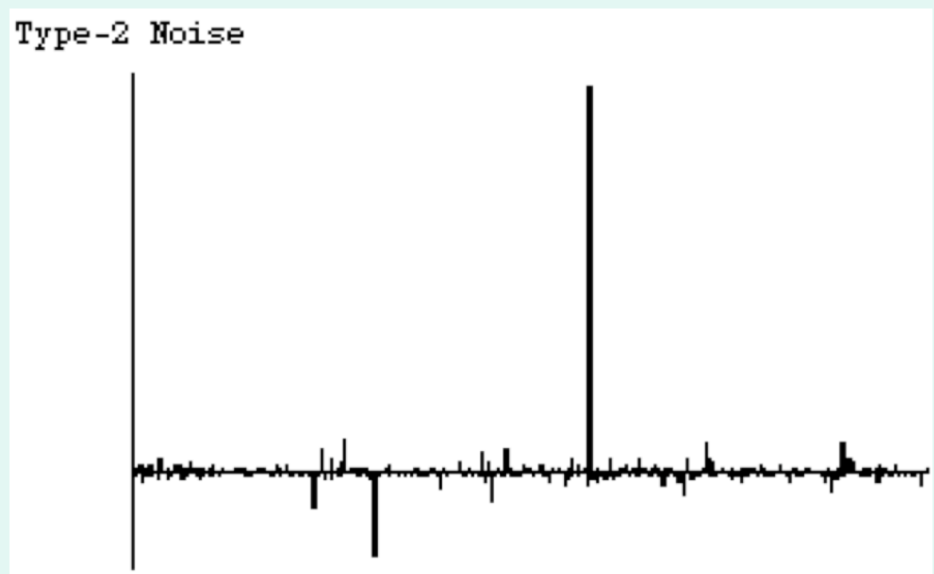
Two Processes





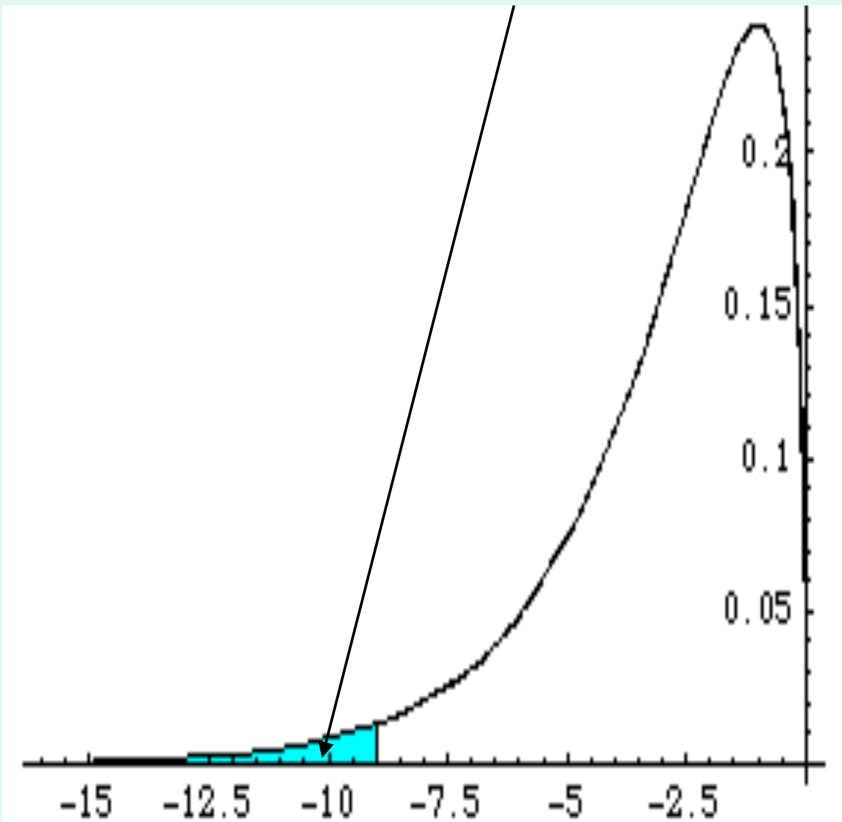

"The Great Moderation"


Type-1 Noise


Type-2 Noise

Left Tail

Invisible Left Tail

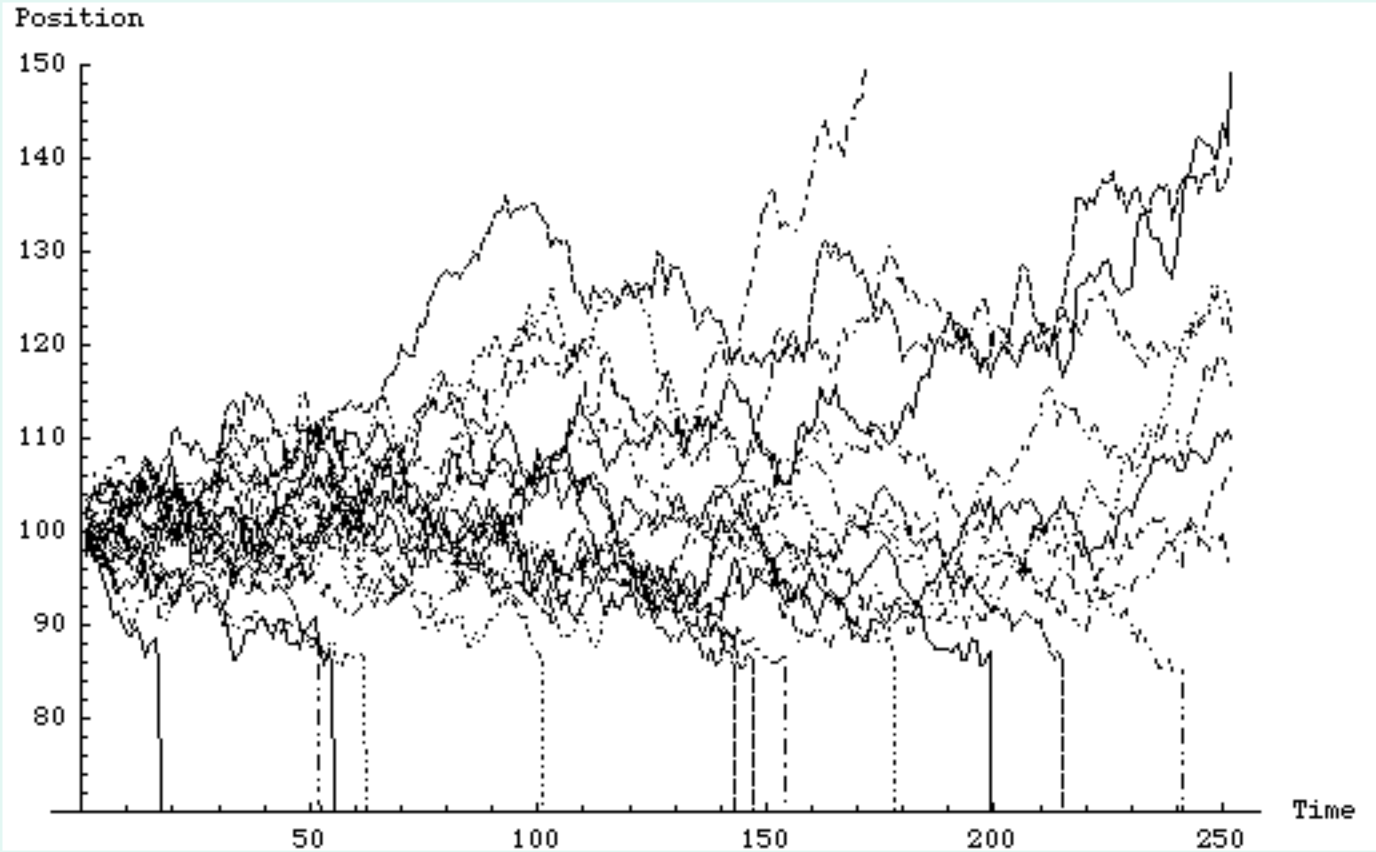


Left Tail & Silent Evidence **(Diagoras problem)**

“conditional on my being here, I didn’t need health insurance”

Bill Fung

Survival Conditioning



Consequence

- Forecasting
- Deficits
- Portfolio theory

Sub-Problems with Small Probabilities

- HARD: Inverse problem (or non-observability of a generator of a random process, degrees of freedom fitting nonlinearities)

(Classical *Problem of Induction*)

- Correcting for survival-conditioned probability
- Preasymptotics

(strong or weak)

- "Atypicality" of Moves
- Correcting for the *Ludic Fallacy*

Central Problem

- HARD: Non measurability of small probability, neither empirically, nor theoretically

Use & Moment $M_m = \sum \pi_i \lambda_i^m$

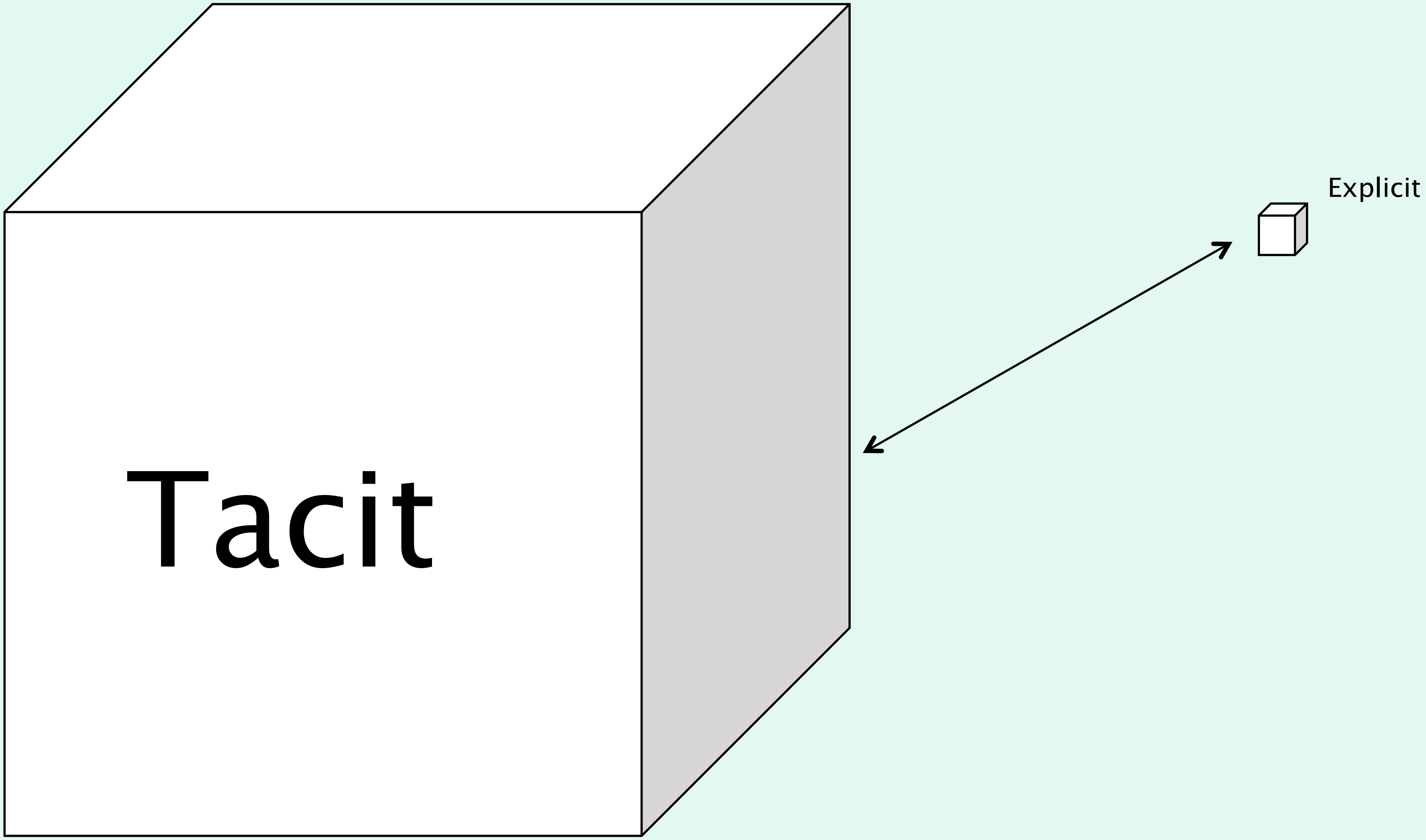
Mo "True/False	**M1 Expectations**	**M2+**
Medicine	Finance (Investments)	Derivative payoffs
Psychology	Insurance	Calibration of nonlinear models
Bets (prediction markets)	General risk management	Kurtosis-based positioning ("volatility trading")
Binary/Digital derivatives	Climate	Cubic payoffs (strips of out of the money options)
Life/Death	Economics (Policy)	
What Else?	Security: Terrorism, Natural catastrophes	
	…About EVERYTHING !	

$$M_m = \int 1_A \, p(\lambda) \, \lambda^m \, d\lambda$$

APPLICATION DOMAIN	Simple payoffs M0 (m=0)	Complex payoffs M1+ (M ≥1)
Distribution 1 (“thin tailed”)	Extremely Robust	Robust
Distribution 2 (no or unknown characteristic scale)	Extremely Robust	LIMITS of Statistics (Black Swan Domain)

More Modest Problem Proposed

- Define boundaries of the Black Swan Domain.
- Program of *Robustness* in the Black Swan Domain


Tacit
Explicit